

A comparison among robust estimation methods for structural equations modeling with ordinal categorical variables

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Abstract

Categorical and ordered variables are commonly used in many scientific researches. Researchers often use the ML method, which assumes a multivariate normal distribution, and this is not true with categorical data because the normal state assumption is violated when a Likert scale is used which leads to shaded results.

In this research, it has been suggested the robust MLR method with covariance matrix of the sample which deals with the data as it is a continuous data especially when the Likert scale is five or above. It has been suggested a method for reducing the error by linking error measurement, where a link was performed between three standard errors, and through the fit indices, it was obtained a good result in reducing the standard error of capabilities and improving the quality of fit indexes. It has been also used two of the robust methods, WLSMV method which known as RDWLS method, and ULSMV method which known as RULS method, use a polychoric correlation, each two methods deal with the data as it categorical. This research also included a comparison between the robust estimation methods ML , MLR , WLSMV and ULSMV and study its effects on the population corrected robust model fit indexes , and then select the best method for dealing with the categorical ordered data . The results showed a superiority of the robust methods in comparison with other methods, where it showed a robust corrections in the standard errors by using the polychoric correlation coefficient matrix, in addition to robust correction of the chi square. In addition of that, the fit indices is replaced by the robust fit indexes of chi- square robust, TLI, CFI and RMSIA.

Keywords: structural equations modeling, robust estimation, Ordered Categorical Variables, fit indices,

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المخلص:

تستخدم المتغيرات الرتبية على نطاق واسع في العديد من البحوث وفي كافة التخصصات العلمية ، وغالبا ما يستخدم الباحثون طريقة الامكان الاعظم ML والتي تفترض التوزيع الطبيعي متعدد المتغيرات ، وهذا ليس صحيحا مع البيانات الرتبية الفئوية حيث ينتهك افتراض التوزيع الطبيعي في حالة استخدام مقياس ليكرات مما يؤدي الى نتائج مضللة وتضخيم في الخطأ المعياري فضلا عن تأثيره على مؤشرات المطابقة.

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فقد تضمن هذا البحث اقتراح طريقة MLR الامكان الاعظم الحصينة مع مصفوفة التغاير للعينة التي تتعامل مع البيانات على انها مستمرة وخاصة عندما يكون مقياس ليكرات خماسي فما فوق. وتم اقتراح طريقة لتقليل الخطأ من خلال ربط الاخطاء ، حيث تم اجراء ارتباط بين ثلاث اخطاء قياسية ، ومن خلال مؤشرات التعديل تم الحصول على نتائج جيدة في تقليل الخطأ المعياري وتحسين جودة المطابقة للمؤشرات.

كما وتم استخدام طريقتين من الطرق الحصينة ، طريقة WLSMV او مايعرف بطريقة RDWLS وطريقة ULSMV او مايعرف بطريقة RULS مع مصفوفة ارتباط متعدد الالوان Polychoric correlation وكلتا الطريقتين تتعامل مع البيانات على انها بيانات رتبية . وتضمن البحث ايضا اجراء مقارنة بين طرق التقدير الحصينة ML ، MLR ، WLSMV ، ULSMV ودراسة تاثيراتها على مؤشرات المطابقة الحصينة ومن ثم اختيار الطريقة الافضل للتعامل مع البيانات الرتبية ، وقد تم التوصل الى نتائج جيدة لكل من الطريقتين ULSMV ، WLSMV مقارنة مع نتائج الطرق الاخرى ، حيث اظهرت النتائج تصحيحات حصينة في الاخطاء المعيارية عن طريق استخدام مصفوفة معامل الارتباط متعددة الالوان بالاضافة الى التصحيحات الحصينة لمؤشر كاي سكوير. فضلا عن ذلك فان مؤشرات المطابقة تم استبدالها بمؤشرات المطابقة الحصينة لكاي سكوير TLI و CFI و RMSIA .

الكلمات المفتاحية: نمذجة المعادلات الهيكلية ، المتغيرات الرتبية الفئوية ، المقدرات الحصينة، مؤشرات المطابقة .

1. introduction

Many scientists or researchers have discussed different estimation methods for modeling structural equations, where Assumed theoretical models include free parameters that we need to estimate, including modeling latent factors, measurement errors, and factor correlations if it is a Measurement model. If the model is structural, the estimated parameters reflect the correlations between basic independent variables and the paths that link the complete independent variables.

The statistical methods that social scientists often use are generally called the first generation techniques. Which involves regression-based approaches such as multiple regression, logistic regression and variance analysis, other tools such as first generation exploratory and confirmatory component analysis, cluster analysis and multidimensional scaling techniques. Nevertheless, many researchers have more increasingly turned to second-generation approaches over the past 20 years to resolve the shortcomings of first-generation methods. researchers introduce non-observable variables or Latent variables that are evaluated indirectly by observed indices. We also make measurement errors in measured variables easier to account for.

2. objective of research

Using robust methods to estimate the parameters in SEM when the data are in ordinal categorical and no normal distribution. Choosing the best way for deal with the ordinal categorical data when we have a five- Likert scale through the estimator that deal with the data as it is a continuous and the robust estimator that deal with the data as it is an ordinal categorical. Study the correcting estimator of robust methods. Measuring the impact of the estimation methods on conformity fit indicators when using the robust Chi Square Correction for Muthen (2010), the corresponding fit indicators for each CFI, TLI, RMSEA are all dependent on the Chi Square robust and are replaced by the new Chi-Square value, the fit indicators are called robust model fit index, and thus it is compared The effect of robust estimation methods on the robust fit indicators. Suggesting a method to reduce the error by making a correlation between the standard errors of the variables, where as the fit indicators give several suggestions to improve the fit of conformity, and a variation was made between covariance $Z62 \sim Z72$ and $Z92 \sim Z102$ and $Y11 \sim Y21$

3. Ordered Categorical Variables

Ordered categorical variable involves more than two categories. Pearson (1901) has a long history of analysis and work of polychoric and polyserial correlations, (Li, Li and Li, 2014). When the data is ordered and categorical, the association measures differ from those for continuous variables. A common definition for ordered categorical variables is that an ordered categorical variable is classified into the observed ordinal variable through applying a number of thresholds. The relationship is called tetrachoric correlation with two underlying continuous variables, while the calculated variables are binary. The resulting correlation is called polychoric association, if the calculated variables have more than two classes.

One way in which observed ordered categorical results occur by dividing a continuous, normally distributed latent response variable (y^*) into different categories (e.g., Bollen, 1989). Thresholds (t) are the points which divide the continuous latent response variable (y^*) into a set number of categories (c) where the total number of thresholds is equal to the number of categories less one ($c - 1$). where $\tau_0 = -\infty$ and $\tau_c = \infty$ is The relationship between a latent response distribution, y^* , with an observed ordinal distribution, y , is formalized as $y = c$, if $\tau_c < y^* < \tau_{c+1}$

The observed ordinal value for y changes if a threshold on the latent response variable y^* is exceeded. For example, if a Likert scale has five response choices, it will require four threshold values to divide y^* into five ordered categories. The ordinal data (y) observed is thought to be

$$y = \begin{cases} 1 & \text{if } y^* \leq \tau_1 \\ 2 & \text{if } \tau_1 < y^* \leq \tau_2 \\ 3 & \text{if } \tau_2 < y^* \leq \tau_3 \\ 4 & \text{if } \tau_3 < y^* \leq \tau_4 \\ 5 & \text{if } y^* > \tau_4 \end{cases} \quad (1)$$

Usually, polychoric correlations are computed using the two-stage method Olsson (1979) defined. (Flora and Curran, 2004) (Course, 2013)

4. Building structural model

The fundamental building blocks of SEM analyzes are implemented using a sequential series of five phases or processes: model definition, model identification, model

estimation, model checking and model adjustment. Such fundamental building blocks are utterly necessary for SEM models to be carried out. Wang (2020)

4.1 Modeling of Structural Equation (SEM)

SEM of two basic sets of models: the measuring model and the model structure, uses the confirmatory factor analysis (CFA) to form the latent variables (factors) and adjust the measuring error of the indicator. The exogenous indicator x measurement model and the endogenous indicators y can be described as

$$\begin{aligned}x &= \Lambda_x \xi + \delta \\y &= \Lambda_y \eta + \varepsilon\end{aligned}\quad (2)$$

The structural model is defined as

$$n = Bn + \Gamma\xi + \zeta \quad (3)$$

where ξ and η are Described as the latent variables vectors given above, B is the $m \times m$ matrix of m^2 Coefficients of regression between the latent endogenous variables, and Γ is the $m \times n$ Coefficients of SR regression matrix among latent endogenous and exogenous variables, ζ is the $m \times 1$ vector with $MVN(0, \Psi)$ observed residual ζ

The covariance matrix is obtained as follows

$$\Sigma(\theta) = \begin{bmatrix} \Lambda_y(I - B)^{-1}[\Gamma\Phi\Gamma' + \Psi](I - B)^{-1}\Lambda_y' + \Theta_\varepsilon & \Lambda_y(I - B)^{-1}\Gamma\Phi\Lambda_x' \\ \Lambda_x\Phi\Gamma'(I - B)^{-1}\Lambda_y' & \Lambda_x\Phi\Lambda_x' + \Theta_\delta \end{bmatrix} \quad (4)$$

Therefore the matrix of covariance was proven. (Timm, no date)

4.2 Estimation of Model Parameters

Estimation is a technique for calculating unknown parameters by optimizing the basic fit function consisting of the hypothesized model and the data observed, Estimation is basically the most important aspect for analysis including methods estimation following

4.2.1 Maximum likelihood function for SEM (ML)

Maximum Likelihood (ML) is the most commonly used fit function for structural equations modeling. Almost every software programs uses ML as their main estimator. This approach, leads to estimates of the parameters which increase the likelihood to obtaining the covariance matrix empirical S . from implied covariance matrix model $\Sigma(\theta)$. The minimized log Likelihood possibility function $\log L$ is (Bollen, 1989)

In this case, MLE function can be defined as in equation (5).

$$F_{ML}(\theta) = \ln |\Sigma| + \text{tr}(S\Sigma^{-1}) - \ln S - (p + q) \quad (5)$$

Where q is X variable number and p is Y variable number. Where θ is a parameter vector. Σ is a covariance matrix model implied. F_{ML} is The fitting function value measured at the estimates final. $|\Sigma|$ determinant. tr is the trace of a matrix. (Bollen, 1989), and Standard errors are the square roots of the diagonal components of the approximate asymptotic covariance matrix from FML under the multivariate normality assumption:

$$\text{acov}(\hat{\theta}) = \left[\varepsilon \left(\frac{\partial^2 F}{\partial \theta \partial \theta' | \theta = \theta_0} \right) \right]^{-1} = n^{-1}(\Delta' \Gamma^{-1} \Delta)^{-1} \quad (6)$$

where $\Delta = (\partial \sigma(\theta) / \partial \theta')|_{\theta = \theta_0}$ Is the model's partial derivatives matrix as respects the parameters. The square roots of the diagonal components then the standard errors. Estimates of parameters provided by ML are desirable asymptotic, such as

unbiased, consistency and efficiency in addition the test statistics which use Wishart's probability are described as

$$T_{ML} = (N - 1)F_{ML}, \quad df = p^* - q, \quad (7)$$

or follows a χ^2 distribution with $p^* - q$ degrees of freedom, where p^* represents the number of non-duplicated elements in the observed covariance matrix $p^* = 0.5p(p + 1)$, where q the number of unknown parameters. (Crisci, 2012) (Bollen, 1989)

4.2.2. Robust Maximum likelihood function for SEM (MLR)

There are two deal methods for non-normal continuous Ordinal data: maximum probability robust (MLR) (Satorra and Bentler, 1994), and weighted least square (WLS) (Browne, 1984). WLS is not advised because its weight matrix requires large sample sizes. MLR is a way of using an asymptotic matrix with covariances. It produces less biased standard errors and works well when dealing with various sample sizes and non-normality degrees. Ordinal measured variables are seldom distributed normally, but often display non-normality in the context of Asymmetrics to a certain degree and showed that in applied studies non-normality in the shape of distribution Asymmetrics (due to Categorical Variables) was very popular.(Micceri, 1989)

Estimates of the parameters derived with ML are not effective asymptotically as long as assumption of normality is not lasting. The Cov()ML in equation (16) not consistent with both asymptotic covariance matrix, resulting in incorrect standard estimates of errors.. (Yuan, Bentler and Zhang, 2005) . Estimated parameters using MLR are similar with those calculated using ML, while the chi-square function and standard parameter-related errors are modified to be robust to non-normal results. If the model is not specified or data is not normal, the correction of SB scaling (Satorra & Bentler, 1994) and (Yuan, Bentler and Zhang, 2005)Asparouhov and Muthén (2005) rescues T_{ML} by

$$T_{ML} = \frac{p^* - q}{tr(U\Gamma^*)} T_M \quad (8)$$

$U\Gamma$ is the weights of the matrix that given by the eigenvalues .

Where $U = W^{-1} - W^{-1}\Delta(\Delta'W^{-1}\Delta)^{-1}\Delta'W^{-1}$, $W = D'(\hat{\Sigma}(\hat{\theta})^{-1} \otimes \hat{\Sigma}(\hat{\theta})^{-1})D$ Is the usual theoretical weight matrix; D is the matrix of duplication; and Γ is either the data kurtosis matrix or a distribution-free approximation of the sample covariance matrix. (Browne, 1984).

There is always a need to rescalue the standard errors. Note that the parameter covariance matrix under the multivariate normality assumption is defined by Equation (16), whereas the robust parameter covariance matrix has a sandwich-like form under non-normality, as shown in the Equation (9) Asparouhov and Muthén (2005)

$$cov(\sqrt{N}\hat{\theta}) = (\hat{\Delta}'W^{-1}\hat{\Delta})^{-1}\hat{\Delta}'W^{-1}\hat{\Gamma}^*W^{-1}\hat{\Delta}(\hat{\Delta}'W^{-1}\hat{\Delta})^{-1} \quad (9)$$

4.2.3. Diagonally weighted squares and Robust DWLS Robust Corrections to Standard Errors and Test Statistics

The WLS estimator's statistical requirements make it an impractical alternative to treat ordered categorical data when an incredibly broad sampling size is accessible (i.e. a complete asymptotic covariance matrix is challenging to quantify and invert). The estimate of Diagonally WLS (DWLS) was developed to address the limitations of full estimate of the WLS. Specifically, by decreasing the statistical sensitivity associated with the complete WLS estimator, DWLS eliminates the need for a large sample size DWLS may also incorporate scaling similar to the S – B scaling approach that results in robust

DWLS estimation or WLSMV (Course, 2013) The general form of the DWLS fit function is:

$$F_{\text{DWLS}} = (S - \sigma(\theta))^T (W_D)^{-1} (S' - \sigma'(\theta)) \quad (10)$$

In ordinary data, one technique is to fit the SEM model with the polychoric correlation matrix rather than the sample covariance matrix called cat-WLS. $W_D = \hat{\text{diag}}(\hat{\Gamma}^*)$ includes only diagonal elements of a polychoric association and threshold projections approximate asymptotic covariance matrix. Therefore, The estimated asymptotic covariance matrix of the parameter calculations provides robust correction of standard errors θ . for D-WLS estimation (Muthén, du Toit, & Spisic, 1997)

$$\text{acov}(\theta)_{\text{DWLS}} = N^{-1} (\Delta' W_D^{-1} \Delta)^{-1} \Delta' W_D^{-1} \hat{\Gamma}^* W_D^{-1} \Delta (\Delta' W_D^{-1} \Delta)^{-1} \quad (11)$$

Asparouhov and Bengt Muthén (2010) proposed a new way to compute the mean- and variance-adjusted χ^2 (denoted as T_{WLSMV}). The method of estimating this correction is called WLSMV or R-DWLS

$$T_{\text{WLSMV}} = aT_{\text{DWLS}} - b \quad (12)$$

$$\text{Where } a = \sqrt{\frac{df}{\text{tr}[(U\hat{\Gamma}^*)^2]}} \text{ , and } b = df - \sqrt{\frac{df[\text{tr}(U\hat{\Gamma}^*)]^2}{\text{tr}[U\hat{\Gamma}^*U\hat{\Gamma}^*]}}$$

4.2.4. Unweighted squares and RULS *Robust Corrections to Standard Errors and Test Statistics*

the ULS is approach of the necessity that all variables observed be on the same scale. One benefit is that the ULS approach does not need a positive-definite covariance matrix, including does not require distributional assumption. (Kline, 2015) (Nalbantoğlu Yılmaz, 2019). Cat-ULS are the approaches that better work in small and medium samples. It is also minimizes squared model residuals; , it uses the matrix of identity as the matrix of weight $W=I$. Recent data indicates parameter estimates for cat-ULS and cat-DWLS is equal with cat-ULS or better performing (Forero, Maydeu-Olivares, & Gallardo-Pujol, 2009); (Yang- Wallentin et al., 2010) (Savalei and Rhemtulla, 2013) . Let r be the $\frac{1}{2}p(p-1) \times 1$ Polychor correlation vector estimated from the categorical data

observed The cat-ULS parameter estimates $\hat{\theta}$ a saturated threshold structure by minimizing the fit can be represented as follows

$$F_{\text{UIS}} = (r - \rho(\theta))' (r - \rho(\theta)) \quad (13)$$

Robust correction of standard errors is taken out in the estimated parameter estimates asymptotic covariance matrix for ULS calculations (Muthén, 1993; Satorra & Bentler, 1994). (Li, 2016)

$$\text{aCov}(\hat{\theta})_{\text{ULS}} = N^{-1} (\hat{\Delta}' \hat{\Delta})^{-1} \hat{\Delta}' \hat{\Gamma}^* \hat{\Delta} (\hat{\Delta}' \hat{\Delta})^{-1} \quad (14)$$

Asparouhov and Muthen (2010) a new approach has been proposed 'to introduce a second order adjustment, one that doesn't change the degrees of freedom of the model. Under this approach, the Robust mean- and variance-adjusted statistics based on the Reliable Cat-ULS estimator are as follows: ULSMV

$$T_{\text{ULSMV}} = aT_{\text{ULS}} - b \quad (15)$$

$$\text{Where } a_{\text{ULS}} = \sqrt{\frac{df}{\text{tr}(\hat{U}_{\text{UIS}} \hat{\Gamma}^* \hat{U}_{\text{UIS}} \hat{\Gamma}^*)}} \text{ , } b_{\text{ULS}} = df - a_{\text{UIS}} \text{tr}(\hat{U}_{\text{UIS}} \hat{\Gamma}^*)$$

(Yang-Wallentin, Jöreskog and Luo, 2010) (Xia and Yang, 2018)

4.3 Model evaluation

A main feature of SEM is the performance of an overall model fit test to the basic hypothesis, $\Sigma(\theta)=\Sigma$, the degree for which the model estimation variance covariance matrix $\hat{\Sigma}$ differs with the sample variance covariance matrix observed S . However , If the model-estimated variance covariance matrix, $\hat{\Sigma}$, is non significantly different with the observed data covariance matrix, S , then we can say the model fits the data well, otherwise, the null hypothesis was rejected . Bollen 1989; Jöreskog and Sörbom 1989; Bentler 1990) The estimation of the all model fit will be performed before the parameter estimates are interpreted. Any assumption from the sample estimation may be misleading without testing the model fit Numerous model fit indices been have developed to determine the closeness of S to Σ . (Bollen,1989)

4.3.1 Comparative fit index (CFI)

(CFI) for Bentler (1990) compares the defined fit model with the null model that assumes no covariances among the observed variables. This estimate is based on the non-centrality parameter $D = \chi^2 - df$ where df is the model's degrees of freedom. as the following format

$$CFI = \frac{d_{null} - d_{specified}}{d_{null}} \tag{16}$$

where d_{null} and $d_{specified}$ are the rescaled non-centrality parameters for the null model and the specified model, respectively , A value of more than 0.90 indicates a good fit. Schumacker and Lomax, 2010)

4.3.2. Tucker Lewis index (TLI)

(TLI) for (Tucker and Lewis, 1973) is also one way to compare the goodness of fit a specified model . and it is defined as

$$TLI = \frac{\left(\frac{\chi^2_{null}}{df_{null}} - \frac{\chi^2_{specified}}{df_{specified}}\right)}{\left(\frac{\chi^2_{null}}{df_{null}} - 1\right)} \tag{17}$$

where $\chi^2_{null} / df_{null}$ and $\chi^2_{specified} / df_{specified}$ ratios of χ^2 statistics to the degrees of freedoms of the null model and ratios of χ^2 statistics to the degrees of freedoms the specified model. A higher value close to one indicates a good fit .Wang (2020)

4.3.3. Root Mean Square Error of Approximation (RMSEA) Index

RMSEA by (Steiger & Lind, 1980) is an indicator of the difference between the covariance matrix with the degree of freedom found , and the assumed covariance matrix indicating the model (Chen, 2007) As for the cut-off limits, the value is 0.08 or less indicates good fit indicators. The fit index is calculated the following way

$$RMSEA_{ML,n} = \sqrt{\max\left(0, \frac{\hat{F}_{ML,n}}{df} - \frac{1}{n-1}\right)} \tag{18}$$

where $\hat{F}_{ML,n}$ indicates the fit function is minimized and n indicates the sample size (Schermelleh-Engel, Moosbrugger and Müller, 2003). In above equation RMSEA provides better results when we increases the sample size compared to the smaller sample sizes . The term $[1/(n - 1)]$ is asymptotically closer to zero when the sample size becomes big (Rigdon, 1996). This test, as described here, is based on a non-centrality parameter:

$$RMSEA_{ML} = \sqrt{\max\left(0, \frac{T_{ML,n} - df}{(n-1)df}\right)} = \sqrt{\max\left(0, \frac{\hat{\lambda}_n}{(n-1)df}\right)} \quad (19)$$

where $\hat{\lambda}_n = T_{ML,n} - df$ is the rescaled non-centrality parameter a CI for The parameter Non centrality is acquired by obtaining the value $\hat{\lambda}_{.05}$ such that $T_{ML;n}$ is the 95th percentile of the chi-square distribution noncentral under; $\chi^2(df, \hat{\lambda}_{.95})$ and $\hat{\lambda}_{.05}$ such that $T_{ML,n}$ is the 5th percentile of the chi-square distribution noncentral under ; $\chi^2(df, \lambda, .05)$ The RMSEA CI limits are defined by

$$RMSEA_{MLn,lower} = \sqrt{\max\left(0, \frac{\hat{\lambda}_{.05}}{(n-1)df}\right)}, RMSEA_{MLn,upper} = \sqrt{\max\left(0, \frac{\hat{\lambda}_{.95}}{(n-1)df}\right)} \quad (20)$$

(Browne and Cudeck 1993) wang (2020) (Brosseau-Liard, Savalei and Li, 2012)

4.3.4. Standardized , Root Mean Square Residual (SRMR)

By (Bentler, 1995) The (SRMR) is an estimate of the standardized average residuals between both the covariance matrices observed and the hypothesized (Chen, 2007). indicates good fit for this indicator is 0.05 or less. They can define as:

$$SRMR = \sqrt{\frac{2 \sum_{i=1}^p \sum_{j=1}^i \frac{(s_{ij} - \hat{\sigma}_{ij})^2}{s_{ii}s_{jj}}}{p(p+1)}} \quad (21)$$

Where s_{ij} is the covariance observed between the two variables , $\hat{\sigma}_{ij}$ represents The corresponding item reproduced in the matrix of covariances, while s_{ii} and s_{jj} are observed standard deviations (Kline, 2011; Schermelleh-Engel and Moosbrugger, 2003)

4.4 Robust Model-fit Indexes with methods robust estimation

As RMSEA, CFI and TLI are all properties of chi-square statistics due to the finite sample sizes, it is conceptually important to replace uncorrected standariz chi-square statistics by robust chi-square statistics when applying them. WLSMV WLSM or ULSMV. The model-fit indexes so defined are called population-corrected (PR) model-fit indexes and are named as RMSEAPR, CFIPR, and TLIPR. (Brosseau Liard et al., 2012)

The chi-square, corrected by mean and variance, is given by $T_{WLSMV} = \frac{1}{a_n} T_{ML} + b$ (Asparouhov & Muthen, 2010) For either WLSMV or ULSMV let T, d, a, and b be the robust chi-square statistics, the degrees for freedom in the model, the scale factor and the shift factor. The design-fit indices of sample size PR are measured as

$$RMSEA_{PR,n} = \sqrt{\max\left(0, \frac{a_H(n-1)\hat{F}_H + b_H - d_H}{(n-1)d_H}\right)} \quad (22)$$

This equation is obtained by simply replacing $T_{ML;n}$ in Equation (19) with T_{SS} . mean- and variance-adjusted muthen (2010) also compute an approximate CI for Equation (20) as follows:

$$RMSEA_{PR,n,lower} = \sqrt{\max\left(0, \frac{\hat{\lambda}_{SS,05}}{(n-1)df}\right)} \quad RMSEA_{PR,n,upper} = \sqrt{\max\left(0, \frac{\hat{\lambda}_{SS,95}}{(n-1)df}\right)} \quad (23)$$

$$CFI_{PR,n} = 1 - \frac{a_H(n-1)\hat{F}_H + b_H - d_H}{a_B(n-1)\hat{F}_B + b_B - d_B} \quad (24)$$

$$TL_{m,n} = 1 - \frac{a_H(n-1)\hat{F}_H + b_H - d_H}{a_B(n-1)\hat{F}_B + b_B - d_B} \cdot \frac{d_H}{d_n} \quad (25)$$

(Brosseau-liard *et al.*, 2014)(Xia and Yang, 2019)

4.5 Modification indices

The indices of modifications help to classify regions of possible model weakness. Their usefulness lies in their capacity to prescribe such changes in order to boost the model's goodness-of-fit. In addition, adjustment indices (provided by all software) will identify the parameters, which greatly contribute to the fit of the model when applied to it. Gana (2019)

5. Applied side

In this part, a comparison is made between estimation methods in terms of parameter estimator , standard error and fit indicators . The model of structural equations is one of the most methods in that used many fields. the model was applied on a data of catigorical ordered from the five Likert scale represented by a questionnaire devoted specified for the the administrative aspect, where the objective of the research is to use the robust estimation methods especially when we deal with the categorical ordered data , so the Violation the assumption of normal distirabuation is predominant. ML is the most common technique available in most programs, Satorra (1998) suggested methods for correcting statistics and standard errors to a degree commensurate with the multivariate kurtosis of the observed data.

An applied study was carried out by relying on data from a doctoral thesis for a field study within the University of Mosul, represented by strategic communication patterns and their reflection in building dynamic capabilities A questionnaire Questions, Mosul University Professors (Ayman, 2019) . Taking part of the scale and constructing a model consisting of 6 latent variables (dimensions) where the latent variable y1 represents the sharing of knowledge. A process by which the organization looks to be creative with the products it provides to its customers. The latent variable y2 brand is the sum total of the banana of the organization that passed to the different audiences of the organization. The latent variable y3 represents the polarization of external knowledge, Gain knowledge from outside the organization (market, research centers, and universities). These three variables operate as latent Exogenous (independent) variables. As for the latent mediation variables, they are represented by Z1: how efficient the integration in which the organization has access to the knowledge that its subsidiaries have. The second latent variable is the mediation Z2, the flexibility of integration represents the multiplicity and type of knowledge areas that the organization possesses and from which it derives its capabilities, while Y1 represents the approved endogenous variable, repair, the organization's ability to learn from its previous experiences and the experiences of other organizations. . the study represented 32 observational variables that represent the paragraphs of the questionnaire distributed among the latent changes, which are not seen. The sample size was 384 views. Modeling requires a sample size greater than 200. One of the well-known rules in the field of determining the least sample size is what Jackson has proposed around the base of q: N , q represents the number of parameters that need to be estimated relative to the sample size for N, and suggested

method two is the number of observe N to the number of variables p , as the sample size is suitable for conducting the study when $10 < (N/p)$ (1/10), i.e. 10 for each variable at least. Jackson(2003)

A Mardia test was performed to verify the assumption of the multivariate normal distribution. find the data do not follow the normal distribution at the level 0.05, also by drawing a QQ-plot. Figure (1) shows that the data are not normal distribution.

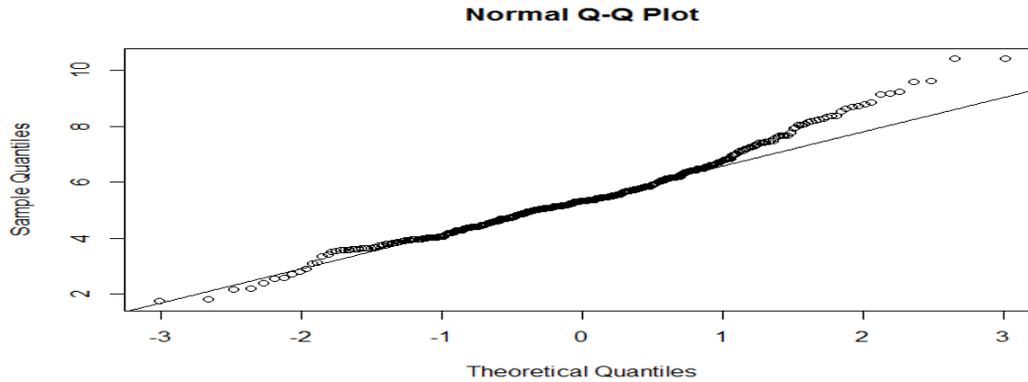


figure (1) chi- squared QQ-plot for data set

The application of transformations in order to change the shape of the distribution into a better approximation to the normal distribution, there are many methods. The interpretation of the transformed variables is often very difficult and the results in wrong conclusions in the medical, administrative and educational fields, and therefore requires the use of alternative methods to deal with non normal distributions. There are several tests that were performed until it was determined before the model was built, by it omitted the non-significant observed variables on the latent variable to construct the model correctly. In addition, verifying for not having been found a problem of multiple linear relationship by the absence of a high correlation between the latent variables. and The mean of all factors is greater than 0.70. The following results are shown by alpha-Cronbach's and omega test and the results are shown in the table (1) and figure (2) Confirmatory Factor Analysis

Table(1) reliability values for on factors

	X1	X2	X3	Z1	Z2	Y1
alpha	0.8578668	0.8691948	0.8922236	0.8831467	0.8552136	0.8799359
omega	0.8600011	0.8698286	0.8928239	0.8840544	0.8549437	0.8797867
omega2	0.8600011	0.8698286	0.8928239	0.8840544	0.8549437	0.8797867
omega3	0.8609672	0.8694482	0.8939084	0.8841796	0.8537930	0.8781420

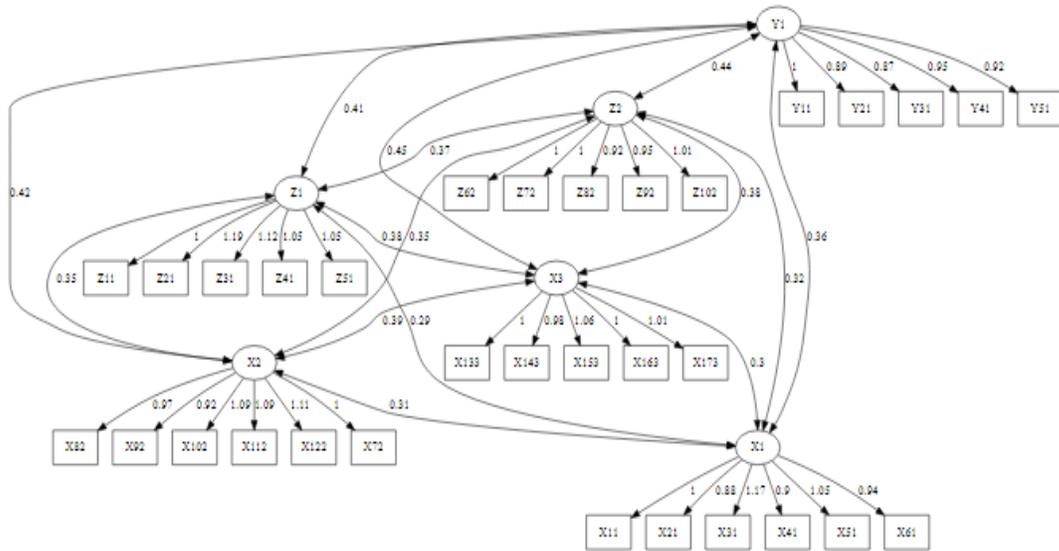


figure (2) Confirmatory Factor Analysis and correlation coefficients between factors

Figures on one-way arrows indicate the values of the estimated parameters of the variables, and the two-headed arrows indicate the correlations between the variables, and the numbers on the arrows indicate the values of the correlations

The hypotheses of modeling the structural equations were developed, the figure(3) showing a path analysis between the latent variables and the use of the R program with the Lavaan package to estimate four methods, namely ML, MLR, ULSMV and WLSMV to find the best method estimation for SEM . as the number of free parameters that required to estimate are 75 parameters which Are variations, covariance , and path analysis regression coefficients. The assumptions for modeling structural equations are shown in the scheme below

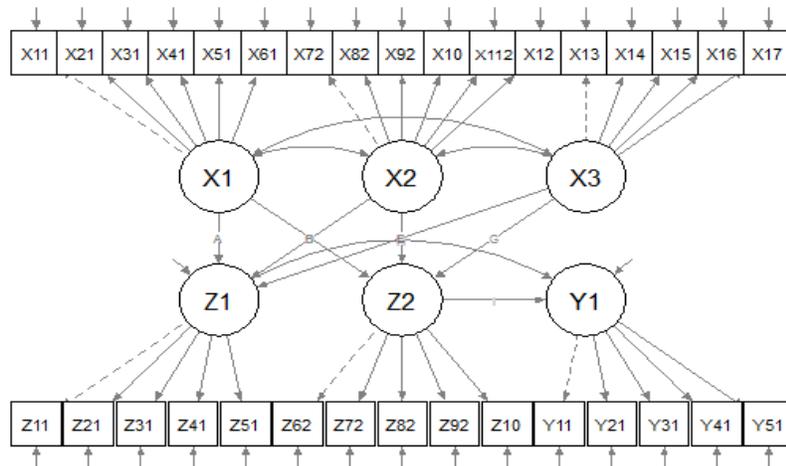


figure (3) Experimental research hypothesis (structural equation model)

The model consists of two parts, measurement the model, which is represented by the following mathematical equations

$$\begin{aligned}
X_{11} &= \lambda x_{11} X_1 + \delta_1 & x_{163} &= \lambda x_{163} X_3 + \delta_{16} \\
X_{21} &= \lambda x_{21} X_1 + \delta_2 & x_{173} &= \lambda x_{173} X_3 + \delta_{17} \\
X_{31} &= \lambda x_{31} X_1 + \delta_3 & Z_{11} &= \lambda z_{11} Z_1 + \delta_1 \\
X_{41} &= \lambda x_{41} X_1 + \delta_4 & Z_{21} &= \lambda z_{21} Z_1 + \delta_2 \\
X_{51} &= \lambda x_{51} X_1 + \delta_5 & Z_{31} &= \lambda z_{31} Z_1 + \delta_3 \\
X_{61} &= \lambda x_{61} X_1 + \delta_6 & Z_{41} &= \lambda z_{41} Z_1 + \delta_4 \\
X_{72} &= \lambda x_{72} X_2 + \delta_7 & Z_{51} &= \lambda z_{51} Z_1 + \delta_5 \\
X_{82} &= \lambda x_{82} X_2 + \delta_8 & Z_{62} &= \lambda z_{62} Z_2 + \delta_6 \\
X_{92} &= \lambda x_{92} X_2 + \delta_9 & Z_{72} &= \lambda z_{72} Z_2 + \delta_7 \\
X_{102} &= \lambda x_{102} X_2 + \delta_{10} & Z_{82} &= \lambda z_{82} Z_2 + \delta_8 \\
X_{112} &= \lambda x_{112} X_2 + \delta_{11} & Z_{92} &= \lambda z_{92} Z_2 + \delta_9 \\
X_{122} &= \lambda x_{122} X_2 + \delta_{12} & Z_{102} &= \lambda z_{102} Z_2 + \delta_{10} \\
x_{133} &= \lambda x_{133} X_3 + \delta_{13} & Y_{11} &= \lambda y_{11} Y_1 + \delta_1 \\
x_{143} &= \lambda x_{143} X_3 + \delta_{14} & & \dots \\
x_{143} &= \lambda x_{143} X_3 + \delta_{14} & Y_{51} &= \lambda y_{51} Y_1 + \delta_5
\end{aligned} \tag{26}$$

As for the structural model, it is written in the following format

$$\begin{aligned}
Z_1 &= \gamma_{11} X_1 + \gamma_{12} X_2 + \gamma_{13} X_3 + \zeta_1 \\
Z_2 &= \gamma_{21} X_1 + \gamma_{22} X_2 + \gamma_{13} X_3 + \zeta_2 \\
Y_1 &= \beta_{11} Z_1 + \beta_{12} Z_2 + \zeta_3
\end{aligned} \tag{27}$$

The parameters $\lambda x_{11} \dots y_{51}, \gamma_{11} \dots \gamma_{13}, \beta_{11} \dots \beta_{12}$ are unknown and their estimation is required. The factor loads of the standard model, the measurement errors on the measured variable, and the structural model parameters represent a pathway analysis between the latent variables. After the assumptions have been set for the model and the measurement and structural model is determined, the estimation process is the most important stage in the modeling, as it is related to the fit function which is reducing the difference between the sample matrix S and the matrix derived by the model. The estimation methods provide us two type of information, the first one estimating the free parameters of the model and standard errors for these estimates, the second is the fit feature between the two matrices, which allows the calculation of good fit indicators.

Traditionally when the Likert scale is five it treats with the data as it is continuous when using both ML and MLR methods, so that we use a Pearson correlation coefficient with these methods. and with development, it suggested several methods to deal with the class data ordered categorical, including the robust methods of each of WLSMV ULSMV using poly correlation coefficient.

A good decision regarding the estimation method has a direct impact on the results, and the ML method does not give biased results when the number of categorical is high and the size is large and the data is distributed almost normality. tables (2) estimate the parameters directly and indirectly. Direct via mediation variables as well as estimates of parameters of the standard model in equation (26) and estimates of parameters of Exogenous, intermediate, and endogenous underlying latent variables of the four methods.

Table (2) Estimation of the parameters of the ML, MLR, WLSMV and ULSMV method for the structural model, standard errors

parameter	methods estimation							
	ML		MLR		WLSMV		ULSMV	
	estimate	Std.Err	estimate	Std.Err	estimate	Std.Err	estimate	Std.Err
X1 =~ X11	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
X1 =~ X21	0.876	0.075	0.876	0.064	0.923	0.054	0.910	0.058
X1 =~ X31	1.166	0.090	1.166	0.081	1.046	0.052	1.046	0.057
X1 =~ X41	0.902	0.074	0.902	0.078	0.996	0.061	0.998	0.068
X1 =~ X51	1.057	0.079	1.057	0.080	1.122	0.059	1.132	0.065
X1 =~ X61	0.937	0.080	0.937	0.082	1.159	0.062	1.181	0.068
X2 =~ X72	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
X2 =~ X82	0.983	0.071	0.983	0.061	1.051	0.037	1.042	0.038
X2 =~ X92	0.924	0.074	0.924	0.078	0.945	0.047	0.945	0.048
X2 =~ X102	1.088	0.078	1.088	0.075	1.046	0.042	1.040	0.043
X2 =~ X112	1.086	0.081	1.086	0.085	1.011	0.042	0.998	0.098
X2 =~ X122	1.110	0.083	1.110	0.083	0.960	0.042	0.949	0.042
X3 =~ X133	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
X3 =~ X143	0.977	0.057	0.977	0.042	1.015	0.027	1.008	0.029
X3 =~ X153	1.053	0.062	1.053	0.057	1.005	0.029	1.002	0.033
X3 =~ X163	0.988	0.060	0.988	0.058	1.011	0.030	1.013	0.034
X3 =~ X173	1.006	0.063	1.006	0.057	0.987	0.029	1.002	0.033
Z1 =~ Z11	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
Z1 =~ Z21	1.19	0.082	1.19	0.082	1.128	0.041	1.127	0.043
Z1 =~ Z31	1.122	0.076	1.122	0.082	1.168	0.043	1.173	0.046
Z1 =~ Z41	1.062	0.074	1.062	0.076	1.076	0.041	1.064	0.043
Z1 =~ Z51	1.062	0.074	1.062	0.079	1.159	0.045	1.164	0.048
Z2 =~ Z62	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
Z2 =~ Z72	1.009	0.070	1.009	0.047	1.003	0.034	1.001	0.036
Z2 =~ Z82	0.929	0.073	0.929	0.060	0.879	0.037	0.905	0.040
Z2 =~ Z92	0.957	0.067	0.957	0.076	1.025	0.039	1.050	0.046
Z2 =~ Z102	1.017	0.070	1.017	0.077	1.025	0.039	1.084	0.046
Y1 =~ Y11	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
Y1 =~ Y21	0.888	0.061	0.888	0.046	1.131	0.038	1.151	0.043
Y1 =~ Y31	0.875	0.059	0.875	0.050	1.121	0.037	1.149	0.043
Y1 =~ Y41	0.945	0.064	0.945	0.060	1.129	0.040	1.156	0.046
Y1 =~ Y51	0.925	0.063	0.925	0.060	1.097	0.042	1.115	0.049
Z1 ~x1	0.101	0.045	0.101	0.059	0.108	0.042	0.094	0.043
Z1 ~x2	0.498	0.084	0.498	0.109	0.511	0.059	0.536	0.063
Z1 ~x3	0.283	0.062	0.283	0.078	0.296	0.044	0.285	0.048
Z2 ~x1	0.187	0.054	0.187	0.068	0.210	0.049	0.207	0.050
Z2 ~x2	0.469	0.094	0.469	0.122	0.478	0.069	0.479	0.071
Z2 ~x3	0.272	0.072	0.272	0.091	0.259	0.055	0.249	0.058
Y1 ~ Z1	0.556	0.094	0.556	0.136	0.487	0.055	0.470	0.062
Y1 ~ Z2	0.525	0.087	0.525	0.131	0.380	0.050	0.399	0.060
dir_Z1	0.556	0.094	0.556	0.136	0.487	0.055	0.470	0.062
dir_Z2	0.525	0.087	0.525	0.131	0.380	0.050	0.399	0.060
ind1_X1_TO_Y1	0.056	0.026	0.056	0.037	0.052	0.021	0.044	0.022
Ind2_X1_TO_Y1	0.098	0.032	0.098	0.045	0.080	0.022	0.083	0.024
tot_X1_TO_Y1	0.154	0.041	0.154	0.060	0.132	0.030	0.127	0.031
ind1_X2_TO_Y1	0.277	0.062	0.277	0.087	0.249	0.039	0.251	0.043
Ind2_X2_TO_Y1	0.246	0.062	0.246	0.088	0.182	0.036	0.191	0.040
tot_X2_TO_Y1	0.523	0.079	0.523	0.113	0.431	0.048	0.442	0.051
ind1_X3_TO_Y1	0.157	0.042	0.157	0.058	0.144	0.026	0.134	0.028
Ind2_X3_TO_Y1	0.143	0.043	0.143	0.057	0.099	0.024	0.099	0.027
tot_nd_4_TO_Y1	0.300	0.056	0.300	0.078	0.243	0.033	0.233	0.035

All Std.Err values are small for all estimators, but there is a difference between the estimators. using Robust corrections for the standard errors leads to a reduction in the errors of the estimator for all parameters . in addition , most of the estimated parameters are greater than twice the standard error , and the sum of the parameter divided by the estimated error is greater than 1.96 which indicates that the parameters are significant. Through the results of the tables above, the MLR method provided better performance than the ML when we deal with the data as continuous using the Pearson correlation coefficient, also , the MLR method presented small standard errors compared to the ML,

where as the estimation method is the same but the correction in the robust standard errors As a result, the corresponding fit indicators provided a perfect match compared with the way ML method, so it is preferable to use MLr with the ordered categorical data that does not normal distribution

We also note from the table of estimators WLSMV, ULSMV robust, a significant improvement in the values of parameter estimates and standard errors. Where as the errors less than methods ML, MLR using polycoric correlation coefficient with wlsmv,ulsmv. Although small results were obtained for standard errors for each estimator by wlsmv, ulsmv, but fit indicators for a ulsmv provided better performance than a wlsmv. Based on the results of the above methods we recommend to use the ULSMV method , for this reason also will be explained the robust ulsmv estimation method in reserch.

By analyzing the results of the model and setting research hypotheses based on theory, there is an indirect effect of the latent Exogenous variables through the mediation latent variables on the endogenous latent variable , and there is no direct effect on the relationship , and there is complete mediation as we note through the application.

Table (2) shows parameter estimates for the ULSMV estimator as there is a direct effect from the Exogenous latent variable for each x_1 x_2 x_3 which represented by knowledge and brand sharing and knowledge polarization on the mediation variable Z1 the adequacy of integration .also, there is an effect on the second mediation variable flexibility of integration Z2 , and all the track effects were significant, achieving the results of the model hypothesis. Also, there was a direct effect by the two mediation variables, Z1 and Z2, on the endogenous variable, Y1 learning.

Through the two mediation variables, there is an indirect provocation of the Exogenous latent variables X1 X2 X3 by the mediation variable Z1, and at the same time there is an indirect effect from the Exogenous latent variables X1 X2 X3 to the endogenous latent variable Y1 by the second mediation variable Z2 ,so that the amount of indirect effect X1 to Y1 by the mediation variable Z1 is 0.44 with a standard error of 0.22.

There is an indirect effect from the variable X1 to Y1 via the second mediation variable variable Z2 which is 0.83 and with a standard error of 0.24 ,while the overall effect of X1 across each of the two mediation variables Z1 Z2 to Y1 is 0.127 with a standard error of 0.31. in the same Method, the effect of the direct and indirect pathway of both X2 to Y1 was studied by the two mediation potential variables Z1 Z2, as well as X3 to Y1 via Z1 Z2 where as all values were significant and errors were small.

5.1 Classical and robust fit indices

The main types of fit indicators were presented, and the assumed sem model was examined from the perspective of different estimation methods. We note that the model estimated according to ML methods obtained good fit indicators, while the RMSEA TLI CFI SRMR indicators was within the ideal interval, and the model estimated under the MLR method obtained higher quality fit indicators than the ML, especially when using the Yuan- Bentler, and the scaling correction factor was 1.218. By dividing this value on the standard Chi Square value of ML we get the robust corrected value which is 834.945, and since the fit indicators for RMSEA TLI CFI depend on the chi-Square corrector, it replaced the value of the robust chi-Square and leads to an improvement in the fit indicators of the conformity.

As for the conformance fit indices of the WLSMV method using the robust Chi Square Correction Factor for Muthén 2010, when we deal with the data categorical ordinal and the polycoric correlation coefficient, the value of Chi Square is 1127.826, while the correction value was equal to Scaling correction factor = 248.365 and shift parameter is 0.971. the fit indicators for the ULSM estimator with Muthén correction 2010, provided superior performance in model fit for all conformance indicators when we deal with categorical data. therefore, we recommend using the ULSMV estimator when the data is ordinal with Likert scale categorical data, contrary to what most researchers use with Common ML estimator in most programs.

From this results, we conclude that the best fit of data when we deal with the data as it is continuous using MLR robust, where as the robust estimator provides a correction in the kurtosis of resulting from the lack of a normal distribution of data, and most of the fit robust indicators performed better than the ML fit indicators, As for the WLSMV ULSMV estimators, the strong fit indicators for the ULSMV estimator provided an optimal fit performance better than the WLSMV when dealing with the data as ordered categorical by correction in the mean and variance. table(4) shows the fit indicators for the methods.

Table (3) indicators of classical and robust fit of the four estimators

estimator	Chi Square	df	χ^2 $\backslash df$	RMSEA	lower	upper	SRMR	CFI	TLI
ML	1016.830	453	2.244	0.057	0.052	0.062	0.047	0.924	0.917
MLR	834.945	453	1.843	0.047	0.042	0.051	0.047	0.935	0.929
WLSMV	1127.826	453	2.489	0.062	0.058	0.067	0.045	0.954	0.950
ULSMV	997.628	453	2.202	0.056	0.051	0.061	0.045	0.955	0.951

5.2 fit indicators of classic and robust fit after adjusting for errors between observed variables

We note through the fit indicators before and after making the covariance between measurement errors Z62 ~ Z72 and Z92 ~ Z102 and Y11 ~ Y21, there is improvement in all indicators for all methods, as the values of the Chi Square have decreased and the values of the root mean square error of approximation index decreased close to 0.05 and less. this indicates that the index is within the good interval, as the closer to zero the greater the strength of fit to the model and the value falls within the interval of confidence accepted. In addition to that, it has been shown increasing in the values of CFI and TLI indicators and its approached one. the value of the SRMR index which is based on the analysis of the standard residual matrix, when ever close to zero indicates a good match and less influence with the parameters of the chi-Square.

Table (4) indicators of classical and robust fit of the four estimators after Adjustment

estimator	Chi-square	df	χ^2 $\backslash df$	RMSEA	lower	upper	SRMR	CFI	TLI
ML	892.994	450	1.984	0.051	0.046	0.055	0.045	0.940	0.934
MLR	734.921	450	1.633	0.045	0.039	0.051	0.045	0.953	0.948
WLSMV	1007.141	450	2.238	0.057	0.052	0.062	0.043	0.962	0.958
ULSMV	941.117	450	2.091	0.053	0.049	0.058	0.043	0.963	0.959

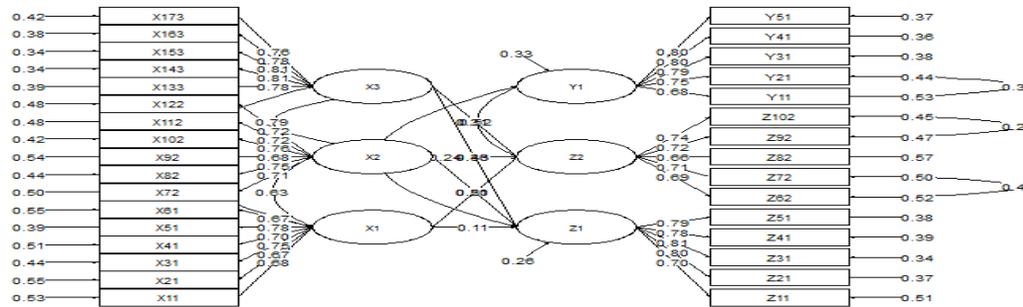


figure (4) adjusting for errors and correlation between observed variables

6. Conclusions

the fit indicators for the MLR provided performance and fit higher than the ML due to the procedures for corrections robust on both the standard errors and the fit index test yuan.bentler. The ULSMV, WLSMV method presented small standard errors compared to the MLR robust when dealing with the data as ordinal Categorical using the polycoric correlation coefficient, , as well as the fit index robust that is used in WLSMV and ULSMV estimators relative to the robust Muthen (2010) gives a good fit. after making the covariance between measurement errors $Z62 \sim Z72$ and $Z92 \sim Z102$ and $Y11 \sim Y21$, there is improvement in all indicators for all methods, standard errors were reduced. We recommend the use of robust methods when the data are not normal distributed and ordinal (categorical). When the data is ordinal (categorical), it is preferable to use each of the WLSMV ULSMV methods, and also when we have a Likert scale greater than 4 categories, it is preferable to use the robust MLR estimator

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