

## An Alternative Computational technique to Study the Sensitivity Analysis of SPP

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### Abstract

In this paper, we present a new computational algorithm which can be used to study the sensitivity analysis of GPP and SPP of degree of difficulty greater than zero. To study the sensitivity analysis of GPP (Dinkal & Kohgenberger [2]), one has to use a subroutine to solve a system of linear equation to get the values of optimal primal variables. Our proposed algorithm is thus more effective in that no new subroutines (such as one for computing the inverse matrices  $J_{ij}^{-1}$  are needed). The generalized algorithm which was presented by Mohan & Al-Bayati to study the sensitivity analysis of SPP [1], is same as our proposed algorithm except that in evaluating the new values of the dual optimal variables. The numerical evidence confirm that the new proposed algorithm is very effective.

**Keyword:** sensitivity analysis, GPP, SPP, primal variables, dual variables.

خوارزمية حسابية بديلة لدراسة تحليل الحساسية في مسائل

البرمجة المتعددة الحدود السالبة

الملخص

في هذا البحث تم اقتراح خوارزمية حسابية جديدة بديلة لدراسة تحليل الحساسية في مسائل البرمجة الهندسية وبرمجة متعددة الحدود السالبة من ذوات درجة الصعوبة الموجبة. لحل هذا النوع من المسائل نحتاج إلى استخدام روتين فرعي لحل منظومة من المعادلات الخطية لإيجاد القيمة المثلى للمتغيرات

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في حين أن الخوارزمية المقترحة لا تحتاج إلى حساب معكوس المصفوفة  $J_{ij}^{-1}$  وتختلف عن خوارزمية Mohan & Al-Bayati لحساب القيمة الجديدة للمتغيرات المقابلة المثلى. وقد اثبت دليل النتائج الحسابية كفاءة الخوارزمية المقترحة.

## 1- Introduction

A geometric programming (GP) is a minimization problem of the form

$$\min g_0(x) = \sum_{k=1}^{n_0} u_k(x) = \sum_{k=1}^{n_0} c_k \prod_{i=1}^n x_i^{a_{ij}}$$

$$s.t. \quad g_j(x) \leq 1$$

$$and \quad x > 0 \quad j = 1, \dots, m$$

where the exponents  $a_{ij}$  are arbitrary real numbers, the coefficients  $c_i$  are positive and each  $g_j$  is a posynomial and are called forced constraints [4]. A signomial programming (SP) is a generalization of a geometric programming, which has the form of a GP, but the objective and constraint functions can be signomials i.e. the coefficients  $c_k$  are allowed to be negative [1]. On studying the sensitivity analysis we notice that the Theils work [3] for unconstrained problems as well as the DK algorithm [2] and the generalized DK algorithm [1] are essentially based on the assumptions that for a geometric programming (GP) or signomial programming (SP) problems with degree of difficulty greater than zero it is possible to :

- (i) Express the solutions to the dual constraints in the form :-

$$\delta_i = b_i(0) + \sum_{j=1}^d r_j b_i(j) \quad i = 1, \dots, n \quad (1)$$

where  $\delta_i$  are the dual variables,  $b_i(j)$  are basis for the column space of the matrix  $a_{ij}$ ,  $r_j$  are certain independent variables and  $d$  is the degree of difficulty of the problem.

(ii) The function which gives the optimized parameters  $\delta_j^*$  and  $v(\delta^*)$  in terms of the variable coefficient vector  $c$  are differentiable in an open neighborhood of  $c$ .

Our work is by rewriting the dual variables and the objective function using some independent dual variables  $\delta_{ind.}$  (the variables related to the degree of difficulty), we will find that  $z(\delta)$  is now essentially a function of the variables  $\delta_i, c_i,$  and  $\delta_{ind.}$   $i= m_k, \dots, n_k,$  and so we can write

$$z(\delta) = z(c'_j, \delta_{ind.}) \text{ where } c'_j = \sigma_j c_j \text{ and } \delta_{ind.} = \delta_{ind.1}, \delta_{ind.2}, \dots, \delta_{ind.d} \quad (2)$$

For a given set of values of the variables  $c'_j,$  there is a set of values  $\delta_{ind.1}^*, \delta_{ind.2}^*, \dots, \delta_{ind.d}^*$  for the variables  $\delta_{ind.1}^*, \delta_{ind.2}^*, \dots, \delta_{ind.d}^*$  for which  $z(\delta)$  is maximum. We can therefore write

$$z(\delta) = z(c'_j, \delta_{ind.}^*) \quad (3)$$

where  $\delta_{ind.}$  is determined by the set of equations  $\frac{\partial z}{\partial \delta_{ind.j}^*} = 0 \quad j= 1, \dots, d$

At the point of maxima  $\delta_{ind.}^*$  these equations may be expressed as

$$f_i(c'_j, \delta_{ind.}^*) = 0 \quad i=1, \dots, d \quad (4)$$

Now suppose that the values of the coefficients  $c'_j$  are changed to  $c'_j + \Delta_j$  then the values of  $\delta_{ind.}^*$  at which the new objective function attains maxima will also change to say  $\delta_{ind.}^* + d\delta_{ind.}$  the new point of maxima will satisfy the conditions:

$$f_i(c'_j + \Delta_j, \delta_{ind.}^* + d\delta_{ind.}) = 0 \quad i = 1, \dots, d \quad (5)$$

Now if we suppose that the functions  $f_i$  are continuous functions of the variables  $c'_j$  &  $\delta_{ind.}$  and possess continuous derivatives in an open

neighborhood of  $(c'_j, \delta_{ind.}^*)$  then we can expand (5) by Taylor series

$$f_i(c'_j, \delta_{ind.}^*) + \left[ \Delta_j \frac{\partial f_i}{\partial c'_j} + d\delta_{ind.} \frac{\partial f_i}{\partial \delta_{ind.}} \right] + \dots + \text{higher order terms} = 0$$

where  $i = 1, \dots, d$ . If we now assume that  $\Delta_j$  and  $d\delta_{ind.}$  are small, then we can neglect the higher order terms of this relation to write :

$$\Delta_j \left[ \frac{\partial f_i}{\partial c'_j} \right]_{(c'_j, \delta_{ind.}^*)} + d\delta_{ind.} \left[ \frac{\partial f_i}{\partial \delta_{ind.}} \right]_{\delta_{ind.}^*} = 0 \quad i = 1, \dots, d \quad (6)$$

Because from (4)  $f_i(c'_j, \delta_{ind.}^*) = 0$ . Eqs.(6) is now a system of  $d$  linear equations in  $d$  unknowns  $d\delta_{ind.}$  and has a solution of the form:

$$d\delta_{ind.} = \left[ \frac{\partial f_i}{\partial \delta_{ind.}} \right]_{\delta_{ind.}^*}^{-1} \left[ \frac{\partial f_i}{\partial c'_j} \right]_{(c'_j, \delta_{ind.}^*)} \Delta_j \quad i = 1, \dots, d \quad (7)$$

Provided  $\left[ \frac{\partial f_i}{\partial \delta_{ind.}} \right]_{\delta_{ind.}^*}^{-1}$  exist, i.e. the matrix  $\left[ \frac{\partial f_i}{\partial \delta_{ind.}} \right]_{\delta_{ind.}^*}$  is non- singular.

Once the increment  $d\delta_{ind.}$  are known we get  $\delta_{ind.}^* + d\delta_{ind.}$  as the new values of the vector  $\delta_{ind.}$  at which  $z(c'_j + \Delta_j, \delta_{ind.})$  has the maxima. The maximum values of the remaind dual variables are now getting by the relations between  $\delta_i$  and  $\delta_{ind.}$  using normality and orthogonality constraint in the dual form the problem .

The new maximum value of the objective function as well as the optimal value of the primal variables can now be obtained as in earlier algorithm in chapter three. It may be observed that in the present analysis our basic assumption is that  $z$  is a continuous function of variables  $c'_j, \delta_{ind.}$  and can be continuously differentiated in an open neighborhood of  $(c'_j, \delta_{ind.})$ . This assumption is in no way more severe than the assumption (ii) given

earlier in this section and which are used in Theils ,DK and generalized DK algorithms.

In the next section we present the explicit form of the computational algorithm for this algorithm which may be used to solve practical problems.

## 2- A new Computational Algorithm for Solving SPP

Assuming the SPP and its dual in the standard form as follow

$$\begin{aligned}
 \min. \quad & g_0(x) = \sum_{i=1}^{n_0} \sigma_i c_i \prod_{j=1}^m x_j^{a_{ij}} \\
 s.t. \quad & g_k(x) = \sum_{i=m_k}^{n_k} \sigma_i c_i \prod_{j=1}^m x_j^{a_{ij}} \leq \sigma'_k \\
 & c_i > 0, x_j > 0 \\
 & \text{with} \\
 & \sigma_i \text{ and } \sigma'_k = \mp 1
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 \max. \quad & v(\delta) = \sigma_0 \left[ \prod_{i=1}^n \left( \frac{c_i}{\delta_i} \right)^{\delta_i} \prod_{k=1}^p \lambda_k^{\lambda_k} \right]^{\sigma_0} \\
 s.t. \quad & \sum_{i=1}^{n_0} \sigma_i \delta_i = \sigma_0 \\
 & \sum_{k=0}^p \sum_{i=m_k}^{n_k} \sigma_i a_{ij} \delta_i = 0 \\
 & \text{with} \\
 & n_p = n; \delta_i \geq 0; m_0 = 1; m_1 = n_0 + 1; m_2 = n_1 + 1; \dots, m_p = n_{p-1} + 1 \\
 & \text{and} \\
 & \lambda_k = \sigma'_k \sum_{i=m_k}^{n_k} \sigma_i \delta_i
 \end{aligned} \tag{9}$$

We suppose that the optimal solution to the problem is known . In order to carry out the sensitivity analysis for this problem we now proceed as below :-

**Step (1)** Write

$$z(c'_j, \delta_{ind.}) = \log(v(c'_j, \delta_{ind.}^*)) = \sigma_0 \sum_{i=1}^n \sum_{k=1}^p \sigma_i \delta_i \log\left(\frac{c_i \lambda_k}{\delta_i}\right) \quad (10)$$

And use this to write the system of the equations

$$f_i(c'_j, \delta_{ind.}^*) = \frac{\partial z}{\partial \delta_{ind.i}} = 0 \quad i = 1, \dots, d \quad (11)$$

**Step (2)** Compute  $\Delta \delta_{ind.}$  for changes  $\Delta_j$  in  $c'_j$  by solving the system of the linear equations

$$\Delta_j \left[ \frac{\partial f_i}{\partial c'_j} \right]_{(c'_j, \delta_{ind.}^*)} + d \delta_{ind.i} \left[ \frac{\partial f_i}{\partial \delta_{ind.i}} \right] = 0 \quad i = 1, \dots, d \quad (12)$$

**Step (3)** The new maximum value of the objective function is

$$v'(\delta^*) = v(c'_j + \Delta_j, d \delta_{ind.}^* + d \delta_{ind.})$$

**Step (4)** The new optimal values of the primal is

$$g_o(x') = v'(\delta^*) \quad (13)$$

**Step (5)** The new optimal values of the primal variables are given by solving the system of the following linear equations:

$$(\log x_j^*) = \left[ (a_{ij})^T (a_{ij}) \right]^{-1} (a_{ij})^T (K_i) \quad (14)$$

where

$$K_i = \begin{cases} \log \delta_i^* + \log v(\delta^*) - \log(c_i + dc_i), i = 1, \dots, n_0, \\ \log \delta_i^* + \log \lambda_k^* - \log(c_i + dc_i), i = n_0 + 1, \dots, n, \end{cases}$$

and  $n_0$  is the number of terms in the primal objective function.

### 3- The Effect of the Sensitivity Analysis in GPP

We first apply the new algorithm to solve the following GPP problem (3.1)

$$\min g_0(x) = 2.419x_1x_2x_3 + 95997x_1^{-1.8}x_3x_4^{-4.8}$$

s.t.

$$288670x_1^{-0.875}x_2^{-0.75}x_3^{-1}x_5^{-0.75} \leq 1$$

$$25819x_1^{-0.2}x_3^{-1}x_4^{0.8}x_6^{-1} \leq 1$$

$$0.03866x_5 + 0.03866x_6 \leq 1$$

$$0.0081666x_2^{-1} + x_2^{-1}x_4 \leq 1$$

$$0.0834x_2^{-1} \leq 1$$

$$x_i > 0; i = 1, 2, 3, 4, 5, 6$$

This problem has 2 degrees of difficulty . The representation ( $\delta_{ind.}$ )

for this problem can be written as:

$$\delta_1 = 0.9554 - 0.24115\delta_{ind.1}; \delta_2 = 0.0446 + 0.2411\delta_{ind.1}; \delta_3 = 1 - \delta_{ind.1}; \delta_4 = \delta_{ind.1};$$

$$\delta_5 = 0.75 - 0.75\delta_{ind.1}; \delta_6 = \delta_{ind.1}; \delta_7 = -0.0089 + 0.1518\delta_{ind.1} - \delta_{ind.2};$$

$$\delta_8 = 0.2143 + 0.3571\delta_{ind.1}; \delta_9 = \delta_{ind.2}$$

For the optimal solution

$$\delta_{ind.6}^* = 0.330558$$

$$\delta_{ind.9}^* = 0.005169$$

The equations  $f_i = \frac{\partial z}{\partial \delta_{ind.i}} = 0; i = 1, 2$  for this problem become :

$$f_1 = \frac{\partial z}{\partial \delta_{ind.1}} = -.2411 \times \log(c_1) + .2411 \times \log(.9554 - .244\delta_{ind.1}) + .2411 \times \log(c_2) -$$

$$.02411 \times \log(.0446 + .2411\delta_{ind.1}) - \log(c_3) + \log(c_4) - .75 \times \log(c_5) + \log(c_6) +$$

$$.1515 \times \log(c_7) + .3571 \times \log(c_8) + .5089 \times \log(.2054 + .5089\delta_{ind.1} - \delta_{ind.2}) -$$

$$.1518 \times \log(-.0089 + .1518\delta_{ind.1} - \delta_{ind.2}) - .3571 \times \log(.2143 + .3571\delta_{ind.1}) +$$

$$.25 \times \log(.75 + .25\delta_{ind.1}) - \log(\delta_{ind.1}) + 0.75 \times \log(.75 - .75\delta_{ind.1}) = 0$$

$$f_2 = \frac{\partial z}{\partial \delta_{ind.2}} = \log(c_9) - \log(c_7) + \log(-.0089 + .1618\delta_{ind.1} - \delta_{ind.2}) -$$

$$\log(.2054 + .5089\delta_{ind.1} - \delta_{ind.2}) = 0$$

Suppose we want to study the sensitivity analysis for 10% changes in

$c_1 = 2.419$  so that  $\Delta_1 = 0.2419$  system of equations (6) becomes :

$$\Delta_1 \left[ \frac{\partial f_i}{\partial c'_1} \right]_{(c'_1, \delta^*_{ind.})} + d\delta_{ind.1} \left[ \frac{\partial f_i}{\partial \delta_{ind.1}} \right]_{\delta^*_{ind.1}} + d\delta_{ind.2} \left[ \frac{\partial f_i}{\partial \delta_{ind.1}} \right]_{\delta^*_{ind.2}} = 0$$

This gives

$$-0.0241102 - 4.9239517d\delta^*_{ind.1} + 2.826621951d\delta^*_{ind.2} = 0$$

and

$$2.826621951d\delta^*_{ind.1} - 25.0052455d\delta^*_{ind.2} = 0$$

Solving the above linear system , we get

$$d\delta_{ind.1} = -0.0052363; d\delta_{ind.2} = 0.000591917$$

So at the new optimal solution

$$\delta^*_{ind.1} = 0.3253217; \delta^*_{ind.2} = 0.004577$$

Using step (3) of this algorithm, the maximum value of the objective function is

$$v^*(\delta^*) = v(c'_1 + \Delta_1, \delta^*_{ind.1} + d\delta_{ind.1}, \delta^*_{ind.2} + d\delta_{ind.2}) = 202174.78$$

The new value of the dual variables are

$$\delta^*_{1} = .87693 \quad , \delta^*_{2} = .123067 \quad , \delta^*_{3} = .674677$$

$$\delta^*_{4} = .3253217 \quad , \delta^*_{5} = .506008 \quad , \delta^*_{6} = .325322$$

$$\delta^*_{7} = .035873 \quad , \delta^*_{8} = .330472 \quad , \delta^*_{9} = .004577$$

The new optimal values of the primal variables are now calculated by using the step (5) of the algorithm .

The result are:

$$x^*_{1} = 17523.123 \quad , x^*_{2} = .0834 \quad , x^*_{3} = 45.5918$$

$$x^*_{4} = .075233 \quad , x^*_{5} = 15.7443 \quad , x^*_{6} = 10.1222$$

Similar analysis was carried out for 50% and 100% percent increases in  $c_i$  . The results are presented in table (1).



#### 4- The Effect of the Sensitivity Analysis in SPP

We now apply the new algorithm to solve two SPP which were solved by Mohan & AL-Bayati [1].

Consider the following SP problem (4.1)

$$\min \quad g_0(x) = 2x_1 x_2^{.5} + x_2 x_3^{-1} x_4^{-2} + x_1^{-2} x_2^{-1} x_3^2$$

s.t.

$$x_1 x_2^{.5} x_3 - x_2^{-1} x_4^2 \leq 1$$

$$-.2x_1 x_2 x_3 \leq 1$$

$$x_1 x_2 x_3 x_4 > 0$$

$$c_1 = 2, c_2 = 1, c_3 = 1, c_4 = 1, c_5 = -1, c_6 = .2$$

$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 1, \sigma_5 = \sigma_6 = -1, \sigma'_1 = 1, \sigma'_2 = -1$$

The degree of difficulty of this problem is 1. For the optimal solution of this problem it is found that  $\sigma_0 = 1$

and

$$\delta^*_1 = .6952112, \delta^*_2 = .1047888, \delta^*_3 = .2, \delta^*_4 = .1239443$$

$$\delta^*_5 = .1047888, \delta^*_6 = .419554, \delta^*_7 = .0191555$$

$$\delta^*_8 = \delta^*_{ind.} = .4191554$$

Also  $J(\delta^*) = -39.50795779$   $J^1(\delta^*) = -.025311356$  and for 10% increase in  $C_6 = .2$ , system of equations (6) becomes

$$\Delta_7 \left[ \frac{\partial f_1}{\partial c'_7} \right]_{(c'_7, \delta^*_{ind.1})} + d\delta_{ind.1} \left[ \frac{\partial f_1}{\partial f_{ind.s}} \right] = 0$$

This gives

$$(-.02) \left( \frac{4}{.2} \right) + d\delta_{ind.1} (158.03705) = 0$$

So that  $d\delta_{ind.1} = .0025311$

So at the new optimal solution  $d\delta_{ind.1} = .4216865$ . For this new value of  $d\delta_{ind.1}$ , the new values for this dual and primal problems were computed by using the appropriate steps of our presented algorithm in section 2. The results are given in Table (4.2). Incremental analysis for 50% change in  $c_6$  was also carried out by this new algorithm. The results are given in the same last table.

**Problem (4.2)**

$$\begin{aligned} \min g_0(x) &= -200x_1^{-1}x_2^{-1} - 500x_1^{-1}x_3^{-\frac{2}{3}} - 820x_1^{-1}x_2^{-\frac{2}{3}}x_3^{-3}x_4^{-1} \\ \text{s.t.} \quad &150x_1^{-1}x_2^{-1}x_3^{-1}x_4^{-1} \leq 1 \\ &x_4 + .01x_2x_3x_4 \leq 1 \\ &x_1, x_2, x_3, x_4 > 0 \end{aligned}$$

For this problem

$$\begin{aligned} c_1 = 200, c_2 = 500, c_3 = 820, c_4 = 150, c_5 = 1, c_6 = .01 \\ \sigma_1 = \sigma_2 = \sigma_3 = -1, \sigma_4 = 1, \sigma_1 = 1, \sigma_5 = 1, \sigma'_1 = 1, \sigma'_2 = 1 \end{aligned}$$

The degree of difficulty of this problem is 1. This problem has a negative value for this objective function (i.e.  $\sigma_0 = -1$ ) the optimal solution of this problem was obtained in [1].

At the optimal solution  $\delta_3^* = \delta_{ind.1}^* = .195573$ .

For 10% increase in  $c_3 = 820$ , system (6) becomes

$$(-82) \begin{pmatrix} -1 \\ -820 \end{pmatrix} + 14.512755d\delta_{ind.1} = 0$$

There for  $d\delta_{ind.1} = .006891$

So at the new optimal solution  $\delta_{ind.1}^* = -202463$ . The new optimal values for this problem can now be calculated as earlier. Computations were

also similarly carried out for 50% change in  $c_3$  . The results are presented in Table (4.3).

We used some brief symbols in the tables of this paper and here are the explanation of those symbols:

- ***MB method*** is Mohan and AL-Bayati algorithm for studying the sensitivity analysis in SP problems
- ***N. SPP*** is our new proposed algorithm for studying the sensitivity analysis in GP and SP problems
- ***Inc. An.*** is the method for finding the approximate solution by updating the Jacobian matrix  $J(x)$  for reducing that rounding error obtained by MB method

Table (4.1) sensitivity analysis at changes  $c_1$  in problem (3.1)

Original Model	$C_1 + 10\%$ solution by		$C_1 + 10\%$ solution by			$C_1 + 10\%$ solution by	
	<i>MB method</i>	<i>N. SPP</i>	<i>MB method</i>	<i>N. SPP</i>	<i>Inc. An.</i>	<i>MB method</i>	<i>N. SPP</i>
$\delta_1^* = .875669$	.876867	.876930	.880687	.881981	.880917	.88412	Not allowable by this algorithm
$\delta_2^* = .124331$	.123133	.123067	.119313	.118018	.119082	.115876	
$\delta_3^* = .669442$	.674409	.671677	.690257	.695623	.695623	.691209	
$\delta_4^* = .330558$	.325590	.3253217	.309743	.304376	.308789	.295487	
$\delta_5^* = .502081$	.505807	.506008	.517693	.521717	.518407	.528384	
$\delta_6^* = .330558$	.325590	.325322	.309743	.304376	.308789	.295487	
$\delta_7^* = .036076$	.035883	.035873	.035268	.035061	.035232	.034716	
$\delta_8^* = .332342$	.330568	.330472	.324908	.322993	.324568	.319817	
$\delta_1^* = .005169$	.004608	.0045771	.002817	.002209	.002708	.001206	
$x_1^* = 18054.59$	17519.34	17523.12	15884.41	15954.91	15897.02	14504.07	
$x_2^* = .0834$	.0834	.0834	.0834	.0834	.0834	.0834	
$x_3^* = 44.6911$	45.5918	45.5918	48.6259	48.8261	48.6775	51.6835	
$x_4^* = .075233$	.075233	.075233	.075233	.075233	.075233	.075233	
$x_5^* = 15.5975$	15.7443	15.7443	16.1836	16.3359	13.2106	16.5893	
$x_6^* = 10.2690$	10.1222	10.1222	9.6829	9.5306	9.6559	9.2773	
$g_0 = 185896.41$	202174.8	202174.8	265407.6	267288.3	265828.9	342106.3	

Table (4.2) sensitivity analysis of the SPP problem 4.1 for changes in  $c_6$

Original Model	$C_6 + 10\%$ solution by		$C_6 + 50\%$ solution by		
	<i>MB method</i>	<i>N. SPP</i>	<i>MB method</i>	<i>N. SPP</i>	<i>Inc. An</i>
$\delta_1^* = .693211$	.694557	.694579	.691398	.692047	.691586
$\delta_2^* = .104789$	.105443	.105421	.108602	.107953	.108414
$\delta_3^* = .2$	.2	.2	.2	.2	.2
$\delta_4^* = .123944$	.127217	.127109	.143009	.139763	.142074
$\delta_5^* = .104789$	.105443	.105422	.108602	.107953	.108414
$\delta_6^* = .419155$	.421774	.4216858	.434407	.431811	.43366
$x_1^* = 3.05035$	2.90809	2.91312	2.45953	2.54088	2.48201
$x_2^* = .597132$	.605248	2.58485	.643182	.621344	.63698
$x_3^* = 2.74504$	2.58248	2.58485	2.16714	2.14248	2.11692
$x_4^* = 1.80736$	1.71202	1.713958	1.42487	1.45475	1.4331
$g_0 = 6.78108$	6.514723	6.514724	5.70585	5.70621	5.70587

Table (4.3) sensitivity analysis of the SP problem (4.2) for changes in  $c_3$

Original Model	$C_3 + 10\%$ solution by		$C_3 + 50\%$ solution by		
	<i>MB method</i>	<i>N. SPP</i>	<i>MB method</i>	<i>N. SPP</i>	<i>Inc. An</i>
$\delta_1^* = .7$	.7	.7	.7	.7	.7
$\delta_2^* = .104427$	.097961	.097536	.078481	.069975	.07706
$\delta_3^* = .195573$	.202039	.202464	.221519	.230024	0.22294
$\delta_4^* = 1.0$	1.0	1.0	1.0	1.0	1.0
$\delta_5^* = .704427$	.697961	.697536	.678481	.66975	.67706
$\delta_6^* = .1$	.1	.1	.1	.1	.1
$x_1^* = 12.0687$	11.9716	11.9566	11.6793	11.3834	11.6292
$x_2^* = 4.67255$	4.622	4.6299	4.43619	4.5972	4.4624
$x_3^* = 3.03834$	3.10002	3.10237	3.32258	3.39615	3.33402
$x_4^* = .875615$	.874611	.87298	.871477	.83295	.86516
$g_0 = -5.06606$	-5.162996	-5.16297	-5.51836	-5.51037	-5.513774

### **Summery:**

This work deals with the geometric programming problems and the generalized geometric programming problems (SPP) with full rank exponent matrix  $(a_{ij})$  with degree of difficulty greater than zero and less than type of the constraints. We presents a new algorithm to study the sensitivity analysis based on the same of Dinkel and Kohgenberger outlines with some different concepts. Also we checked our algorithm by solving some test problems.

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